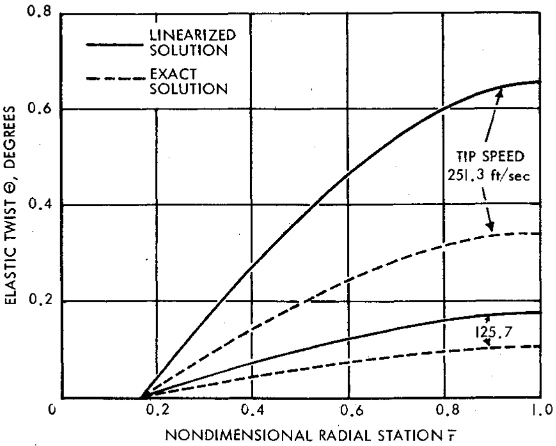

Fig. 4 Comparison of the linearized solution with the exact solution for elastic twist (with $V=0$, $\alpha_r=4°$).

Results obtained by solving the linearized form of Eq. (1) are shown in Fig. 4. The solution of the linearized equation yield higher elastic-twist distribution than the exact solution from Eqs. (1) and (3). Consequently, the inflow velocity distribution is increased and is greater than the corresponding exact solution as discussed previously.

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Airfoil Design by Optimization

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I. Introduction

THE inverse problem of determining an airfoil to support a given pressure distribution is an extremely important problem in aircraft design. This ensures the kind of flow that a designer wishes the wing to support so that criteria like shock delay and separation free zone are achieved at the design condition.

This problem has already been tackled by Lighthill[1] and Thwaites[2] in the forties by conformal transformation methods. In those days electronic digital computers were not available and emphasis was more and more on analytical methods which could not take care of all realistic situations adequately. The present method utilizes an analytical method for representing the airfoil contour by Wagner functions[3] and a computational procedure for obtaining the values of the coefficients of expansion. Lighthill's method could be applied only for a class of airfoils and Thwaite's method[2] is limited by convergence difficulties. The method given in this report is in that sense universal.

Section II gives the representation of an airfoil contour by Wagner functions. Section III validates the procedures for a NACA 0009 airfoil while in Sec. IV it is applied to a typical airfoil which gives a "roof-top" pressure distribution.

II. Representing an Airfoil Contour by Wagner Functions

In an earlier report[6] a procedure was given for approximating an airfoil contour by Wagner functions. Only a few details are given in this section. The slope of an airfoil can be approximated by a Fourier type of series of Wagner functions wherein the coefficients are determined by the method of least squares. Let $y=f(x)$ represent the airfoil contour. Then the slope $f'(x)$ is given by

$$f'(x) = \sum_{n=0}^{\infty} a_n h_n(x) - a_0 \tag{1}$$

where a_0 is the trailing-edge slope and $h_n(x)$ are called Wagner functions[3] defined by

$$h_n(x) = \frac{1}{\pi} \frac{T_{n+1}(1-2x) + T_n(1-2x)}{(x-x^2)^{1/2}} \qquad 0 \le x \le 1 \tag{2}$$

Here $T_n(x)$ are Tychebychev polynomials. Putting

$$x = \sin^2\frac{\theta}{2}, \qquad 0 \le \theta \le \pi \tag{3}$$

we have

$$h_n(\theta) = \frac{2}{\pi} \frac{\cos(\overline{n+1}\theta) + \cos\upsilon\theta}{\sin\theta} \tag{4}$$

First three Wagner functions are given by

$$h_0(\theta) = \frac{2}{\pi}\cot\frac{\theta}{2}$$

$$h_1(\theta) = \frac{2}{\pi}(\cot\frac{\theta}{2} - 2\sin\theta)$$

$$h_2(\theta) = \frac{2}{\pi}(\cot\frac{\theta}{2} - 2\sin\theta - 2\sin 2\theta) \tag{5}$$

The recurrence relation satisfied by $h_n(x)$ is

$$h_n(n) = 2(1-2x)h_{n-1}(x) - h_{n-2}(x) \tag{6}$$

Integrating Eq. (1) and changing the variables from x to θ, we get

$$f(x) \equiv z(\theta) = a_0\frac{(\theta+\sin\theta)}{\pi} + \frac{1}{\pi}\sum_{n=1}^{\infty} a_n\left(\frac{\sin\overline{n+1}\theta}{n+1} + \frac{\sin n\theta}{n}\right) - a_0\sin^2\frac{\theta}{2} \tag{7}$$

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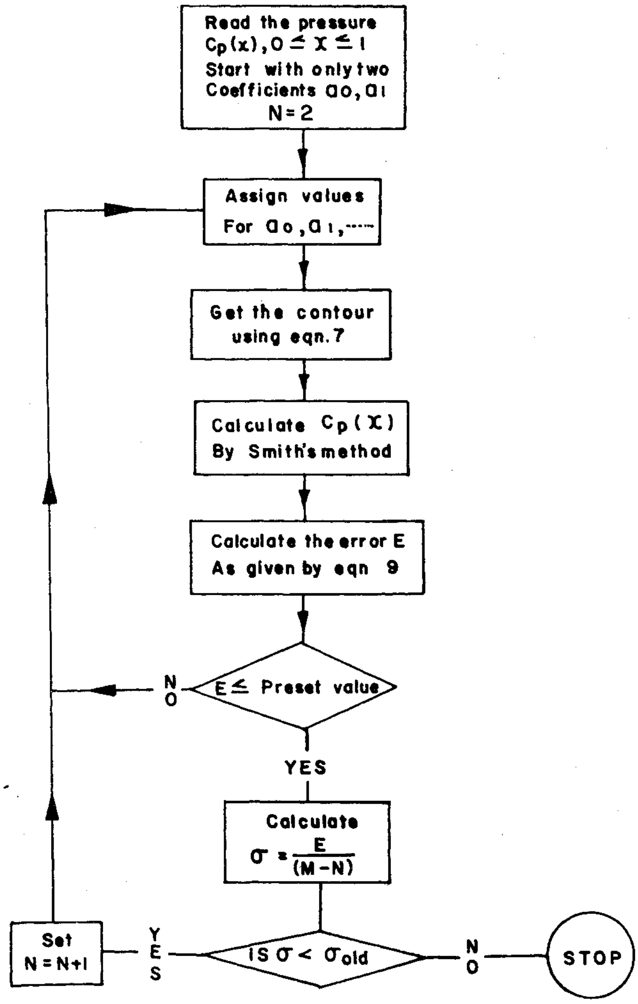

Fig. 1 Flow chart for airfoil design by optimization.

The leading-edge radius ρ is given by

$$\left[\frac{\mathrm{d}z}{\mathrm{d}\theta}\right]_{\theta=0}=\sqrt{\frac{\rho}{2}}=\frac{2}{\pi}\sum_{n=0}^{\infty}a_n \tag{8}$$

The procedure for obtaining the airfoil contour by making use of the expansion (7) is as follows.

Let the pressure distribution $C_p(x)$, $0 \le x \le 1$, be given. We assume the airfoil contour to be represented by an expansion of the type (7). Starting with assumed values of a_0 and a_1, only, the pressure distribution for a symmetrical airfoil at zero angle of attack is obtained by Smith's method.[7] Then the cumulative error function viz.

$$E(a_0,a_1,...)=\int_0^{\pi}[C_p(\theta)-\bar{C}_p(\theta)]^2\sin\theta\mathrm{d}\theta \tag{9}$$

is minimized by an optimized procedure. In Eq. (9) $C_p(\theta)$ and $\bar{C}_p(\theta)$ are given and calculated pressure distributions. This optimization procedure is terminated when the relative error

Table 1 Standard Deviation σ vs number of coefficients N for NACA 0009 and RAE 102-8% airfoils

Airfoil	N = 2	3	4	5
NACA 0009	1.5×10^{-5}	0.4×10^{-5}	0.29×10^{-5}	3.8×10^{-5}
RAE 102−8-%	3.9×10^{-5}	0.22×10^{-5}	0.2×10^{-5}	2.2×10^{-5}

Table 2 Given and calculated values for NACA 0009

X/C	y/C given	y/C calculated
0.0	0.00000	0.00000
0.0125	0.01420	0.01476
0.025	0.01961	0.02013
0.05	0.02666	0.02707
0.075	0.0315	0.03185
0.1	0.03512	0.03545
.15	0.04009	0.04045
0.2	0.04303	0.04343
0.25	0.04456	0.04497
0.3	0.04501	0.04537
0.4	0.04352	0.04367
0.5	0.03971	0.03968
0.6	0.03423	0.03419
0.7	0.02748	0.02761
0.8	0.01967	0.01993
0.9	0.01086	0.01088
0.95	0.00605	0.00569
1.0	0.00000	0.00000

Table 3 Given and calculated values for RAE 102-8% airfoil

X/C	y/C given	y/C calculated
0.00000	0.00000	0.00000
0.0038	0.00574	0.00627
0.0150	0.01138	0.01194
0.0336	0.01679	0.01719
0.0592	0.02188	0.02213
0.0746	0.02426	0.02449
0.1099	0.02867	0.02896
0.1511	0.03249	0.03297
0.1974	0.03563	0.03633
0.2482	0.03801	0.03882
0.3026	0.03951	0.04025
0.3597	0.04000	0.04050
0.4188	0.03919	0.03953
0.4489	0.03820	0.03861
0.5102	0.03541	0.03596
0.5410	0.03370	0.03429
0.6025	0.02984	0.03043
0.6628	0.02562	0.02613
0.7211	0.02129	0.02171
0.7763	0.01709	0.01744
0.8273	0.01320	0.01352
0.8731	0.00969	0.01006
0.9129	0.00668	0.00707
0.9596	0.00301	0.00348
1.0000	0.00000	0.00000

at any stage is less than a preset value. Correspondingly, ; standard deviation σ is defined by the expression $\sigma=(E/M-N)$, where N is the number of coefficients chosen and M is th number of data points at which $C_p(\theta)$ are given. Thi procedure is continued by assuming more and more values o a's and calculating σ at every stage. The procedure is tei minated when σ reached its minimum. Making use of thes coefficients, only the airfoil contour is obtained analyticall and adequately. The procedure is very well illustrated in th flow chart given in Fig. 1.

III. Application to NACA 0009 Airfoil

Table 1 gives the illustration of the above procedure fc NACA 0009 where σ is tabulated against the number (coefficients chosen. It is seen that at $N=4$, σ reach minimum; and with these values of a_0, a_1, a_2, a_3 the e pansion (1) gives the airfoil coordinates. Table 2 gives tl values of the coordinates of NACA 0009, and it is seen th the error between the given and calculated coordinates within 6%.

IV. Application to RAE 102-108% Airfoil

Having validated the procedure against NACA 0009 we applied the same to RAE 102-8% airfoil. This is a typical airfoil which gives a 'roof-top' pressure distribution. Again the comparison between given and obtained coordinates show (Table 2) good agreement. Table 1 gives σ for this airfoil also.

V. Conclusion

A new method for designing an airfoil by using "Wagner expansion" for an airfoil contour and an optimization technique for execution is given. This has the definite advantages compared to classical methods.

Firstly, it does not get into trouble of not closing at the trailing edge like other classical methods for, the expansion (1) itself ensures the closure at the trailing edge.

Secondly, it can be used for any general airfoil unlike the method of Lighthill.

Further improvements in the method of expansion for an asymmetrical airfoil and incorporation of Smith's method for a design C_L will enhance the usefulness of the method for designing an arbitrary airfoil at design C_L whose pressure distribution has been given. This is presently under the investigation by the authors.

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Optimal Trajectories for the Dive-Bombing Mission of a Fighter Aircraft

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Introduction

THE dive-bombing mission of a fighter aircraft is inherently complex as it involves the interaction of the attacking pilot aircaft system and the enemy's target-defense system. The attacking pilot maneuvers his aircraft close to the target for better accuracy in aiming a weapon onto the target, and simultaneously tries to minimize his stay within the enemy's fire envelope for safe return. He has to arrive at a proper tradeoff between the twin objectives of safety and accuracy of target hit. The performance envelope of the aircraft and its handling qualities, the deteriorating effects of normal acceleration and psychological stress on the pilot, any operational limitations arising due to the topology of the target and the surrounding terrain, and the geometry of the target-defense system are some of the important aspects to be considered in this problem.

Recently pilot effects on weapon delivery accuracy in air-to-ground bombing have been studied as a linear stochastic problem.[1,2] However, the determination of optimal trajectories which bring out the best tradeoff between safety and accuracy of target hit, for varying stategies of the target defender is a problem which has not received much attention in the published literature, probably due to its complexity. In this Note we present some of the results obtained so far on this latter aspect.

Dive-Bombing Mission as a Trajectory Optimization Problem

The target-defense system normally consists of a set of radar-controlled anti-aircraft guns stragetically located around the target. The effective zone within which the shell from a gun has a high probability of hitting the attacking aircraft can be modeled as an ellipsoid whose axes represent the horizontal and vertical ranges of the gun.[3] The total fire envelope of the target-defense system is the overall boundary of the individual envelopes which can be approximated as a smooth close surface. Since the radar is ineffective below a certain height, so are the radar-controlled guns.

The usual stragety adopted by the pilots in this mission is to dive towards the target after acquisition by piercing the fire envelope at some entry point, maneuver the aircraft so as to achieve a predetermined weapon release condition (termed pickle point), release the weapon and then fly out of the fire envelope at some exit point. In practice, weapon impact errors arise due to differences in the planned and actual weapon

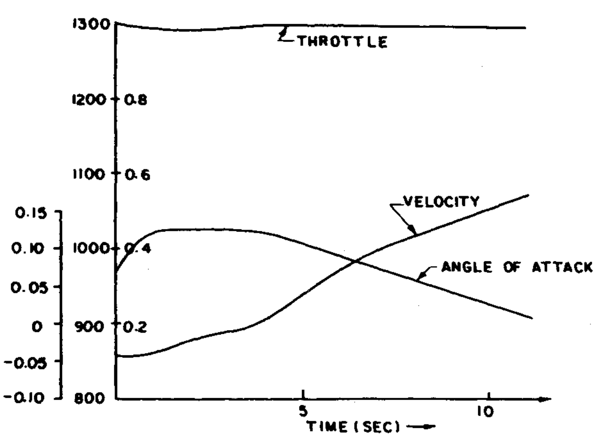

Fig. 1a Velocity and control variable programs (Case 1).

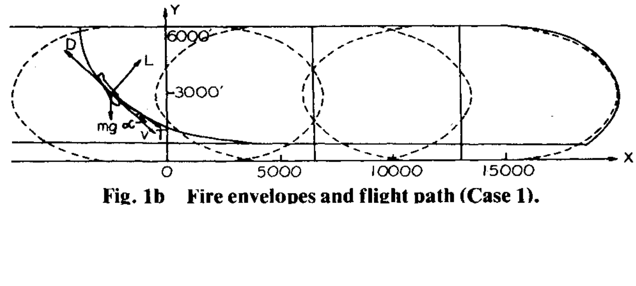

Fig. 1b Fire envelopes and flight path (Case 1).

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